Option Contracts

05/03/2025

Like a futures contract, an option contract provides the right to purchase an asset at a future date with a preset price, not the obligation of fulfilling the contract. Therefore, it is a hedging tool to modulate the gain or loss of a transaction. The contract price is the difference between the current asset price and the discounted preset future price. For a deterministic asset, the call option contract price is:

$$C(r,T) = S_0 - \frac{K}{(1+r)^T}$$

where C is the call option price, r the interest rate, T terms of the contract, S_0 the current asset price, and K the preset price or the strike price of the asset. At the contract closure (exercising contract), the gain or loss is either the difference between the market asset price then and the strike price or nothing if one walks away from the contract. When r is negative, such a contract becomes an inflation hedge.

$$P = \max\{S_T - K, 0\}$$

where P is the payoff of the contract. Therefore, the profit through the transaction is:

$$\frac{P}{C} - 1 = \frac{\max\{S_T - K, 0\}}{C(r,T)} - 1$$

Black-Scholes Option Pricing Formula

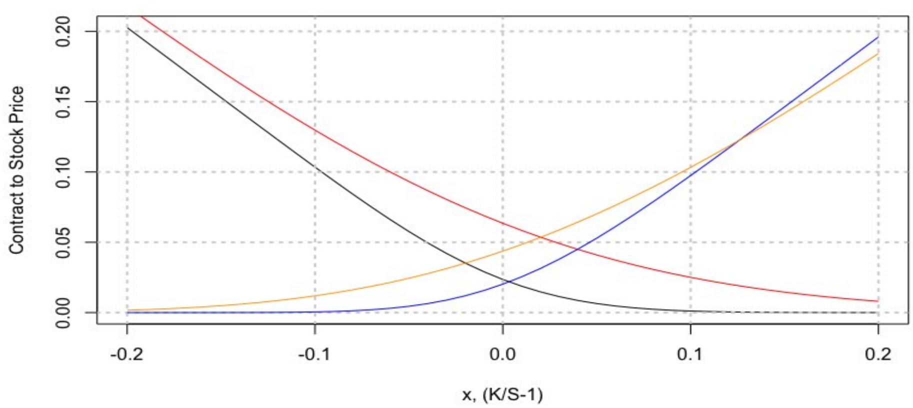

Figure 1. The ratio between the Black-Scholes option price and the stock price as the function of x, (K/S-1). The black and red lines are for the call options and blue and orange lines for put options. The black and blue lines have the term of one month and red and orange lines 6 months.

For an asset whose price follows geometric Brownian motion, the option pricing formula uses a continuous-time stochastic model that yields an explicit solution. For a call option:

$$C = S_0 N(d_1) - K N(d_2) e^{-rT}$$

where N(d) is the cumulative standard normal distribution; d_1 and d_2 are

$$d_1 = \frac{ln\left(S_0/K\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln\left(S_0/K\right) + \left(r - \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

where σ is the annualized standard deviation of the asset return. For a deterministic asset, $\sigma \to 0$, the option price converges to the first formula. Before entering into an option contract, several choices must be made. This article explores these choices on the potential profit of a call option contract. The put option contracts are not considered here.

Underlying Assets

There are many financial assets that offer option contracts. The most traded ones are SPX (SP500) and NAX (NASDAQ). The prices of both are around \$6000 each and the option prices are proportional to the stock prices. In Figure 1, the at-the-money (ATM), 6 month term option price is approximately one twentieth of the stock price. For a \$6K stock, one lot of option contracts (100 shares) costs approximately \$30K. There are certainly more affordable ones. For example, SPY and QQQ are about \$500 each and are proxies to SPX and NAX. A 6-month ATM contract lot (100 shares) of SPY or QQQ can be purchased at \$2.5K each.

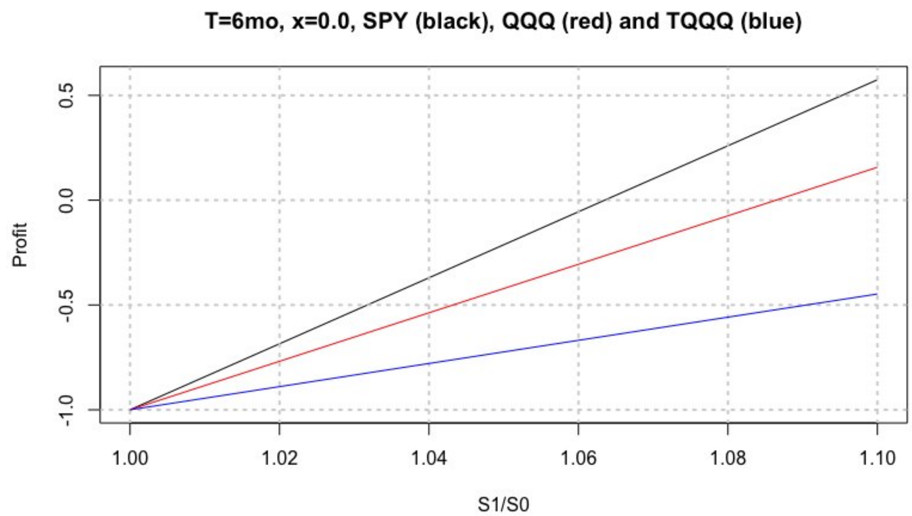

Figure 2. The potential profit as the function of the stock prices ratio of sale and buy, S1/S0, for several Important underlying assets.

Figure 2 shows the profit of ATM call option as function of the underlying stock prices change. SPY is the most responsive to the stock price change and breaks even when the stock increases more than 6%. It further makes 50% profit when the stock price increases another 4% and reaches 10%. On the other hand, TQQQ is less responsive to the stock price change and lost 50% even when the underlying stock increases 10%. Please note that the βs of SPY, QQQ, and TQQQ are 1.00, 1.17 and 3.48, respectively. It is possible that the price change in TQQQ could be as high as 3.5 times of SPY and this would make a positive impact on the option contract.

Option Expiration Terms

The actual option terms are either structured quarterly or monthly. Some highly traded indices even offer weekly option contracts. As shown in Figure 1, the cost for a short term option contract is lower than that of a long term contract; therefore it generates a higher profit for a given underlying stock price change. However, a short term contract leaves the position unprotected for unexpected market gyration in situations where the contract expires before the breakeven is archived. In this case, one could lose 100% of the option contract premium. A longer term contract may cost more but offers a time margin for the market to recover from a sudden price drop. For the example in Figure 1, the cost for a 6-month ATM contract is approximately 3 times of a 1-month contract, but the time margin is 6 times.

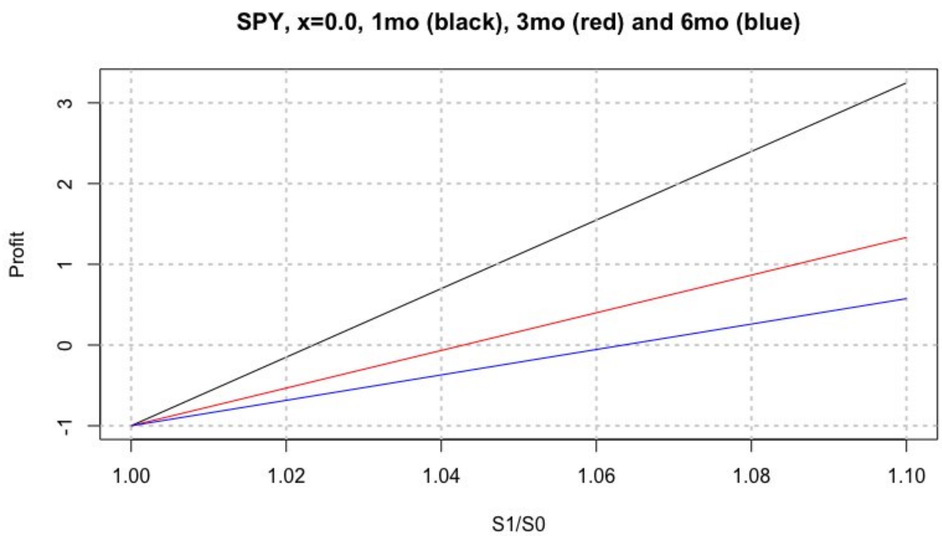

Figure 3. The potential profit as the function of the stock prices ratio of sale and buy, S1/S0, for several terms of the option contract.

Also, the profit of a short term contract is more responsive to the underlying stock price changes than a long term contract. The breakeven point is at 2% stock price increase for a 1-month contract but 4% for a 3-month contract.   Again, one may argue that it is more probable to experience a 10% price move over 6 months than a 3% move over 1 month, both would generate a 50% potential profit.

Strike Price

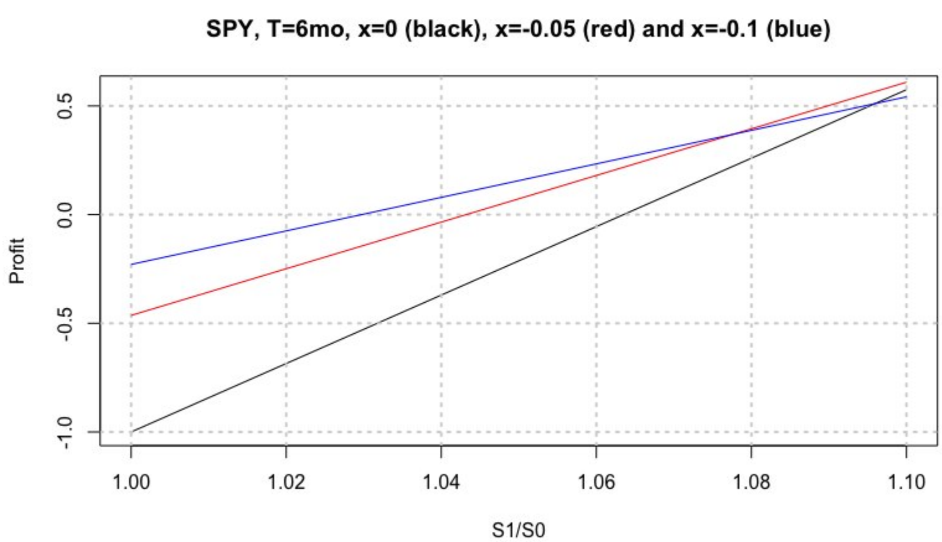

Figure 4. The potential profit as the function of the stock prices ratio of sale and buy, S1/S0, for several strike price ratio of the option contract, where x = K/S – 1.

In-the-money (ITM) contracts (x<0) exhibit weak profit responses to the underlying asset price change but at the same time enjoy lower thresholds for breakeven. Out-the-money (OTM) contracts are not shown in Figure 4, but they are strongly responsive to the underlying asset price but suffer from a higher breakeven price point. The contracts at x=0 are also known as the at-the-money (ATM) contracts.

Concluding Remarks

The profit formula shown previously can be expressed in another form:

$$\frac{P}{C} = \frac{S_1/S_0 - (1+x)}{cs}$$

It is clear that the profit relates to the underlying asset price change (S_1/S_0) linearly with the slope 1/cs and intercept – (1+x)/cs. A large slope and a small intercept are desired. The term 1/cs is also known as the leverage of the option contract whose relation to "money-ness" is the reciprocal of Figure 1. Moreover, the breakeven point of the contract is achieved when P/C = 1. The relationship between breakeven and contract term, T, and degree of "money-ness", x, are illustrated in Figure 5.

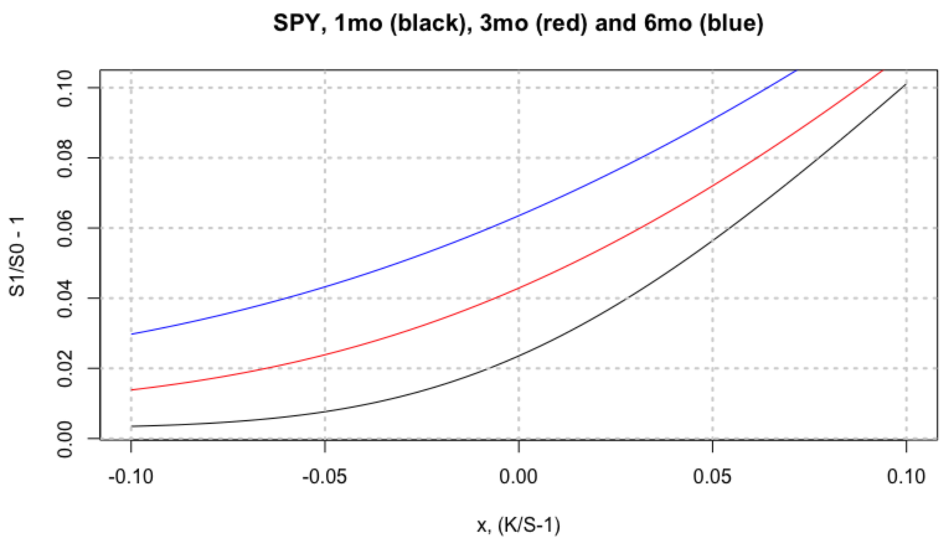

Figure 5. The breakeven point as function of the contract term and the "money-ness", x = K/S – 1. The vertical scale is the fraction of the underlying asset price movement. For example, 0.06 is 6% of price increase from the contract time.

As shown in Figure 1, a short term contract provide a higher leverage of the underlying asset price movement, and an earlier breakeven (Figure 5). A long term contract offer a margin of safety but a smaller leverage and higher breakeven price point. OTM contracts have higher leverages (again, see Figure 1), but also have higher breakeven price points (Figure 5).

Based on these discussions, SPX and SPY can be selected as the trading vehicle and possibly TQQQ may be attempted at certain times. A sound margin of safety guarding against market turbulences must be adopted at all times; therefore a longer term contract is advised with perhaps an aggressive "money-ness" to offset the weak leverage on the asset price movement.