US Gross Domestic Product
12/05/2025

Gross Domestic Product (GDP) measures the market value of everything produced domestically in a country, which acts as a rough measure of a nation's economic activity. The most common way of computing a nation's GDP is to sum up all the spending, including consumption (C), investment (I), government spending (G), and net exports (X-M). It is measured by either the total income generated (production approach) or total income earned (income approach):

$$GDP = C + I + G + (X - M)$$

An online database, Federal Reserve Economic Data (FRED), is created and maintained by the Research Department at the Federal Reserve Bank of St. Louis. The data include quarterly US GDP numbers available at https://fred.stlouisfed.org/. This article uses GDP data downloaded from FRED.

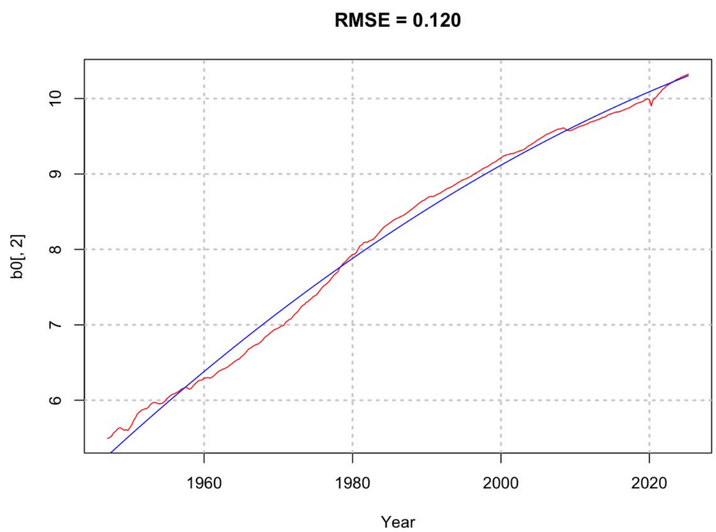

Figure 1, Logarithmic US GDP (red) and the General Trend Line (blue) using a Quadratic Linear Model.

The downloaded data was first transformed into the logarithmic form to help "regularize" the data that span over a large magnitude ($250 billion to $31 trillion). Figure 1 shows the data and a quadratic linear fit. A higher order linear model does not improve the root mean square error (RMSE). The overall trend shows that the US GDP growth is slowing down, especially after year 2000 with short-term ups and downs. The seasonality of the data is modeled using the Fourier transformation. Figure 2 shows the spectrogram of the residual term from the trend model.

An important frequency component is located at a frequency index of 3 corresponding to a seasonality of 314/3 = 104 quarters (26 years). The Fourier transformation therefore includes all the frequency components whose index is below 6. Figure 3 shows the modeling result. The RMSE decreases from 0.12 to 0.029, a remarkable improvement. This 26-year cycle perhaps describes the slow down in 1960s, a rapid growth in 1970s, and another rapid growth after 2020. This "wavy" growth pattern becomes more pronounced in Figure 3.

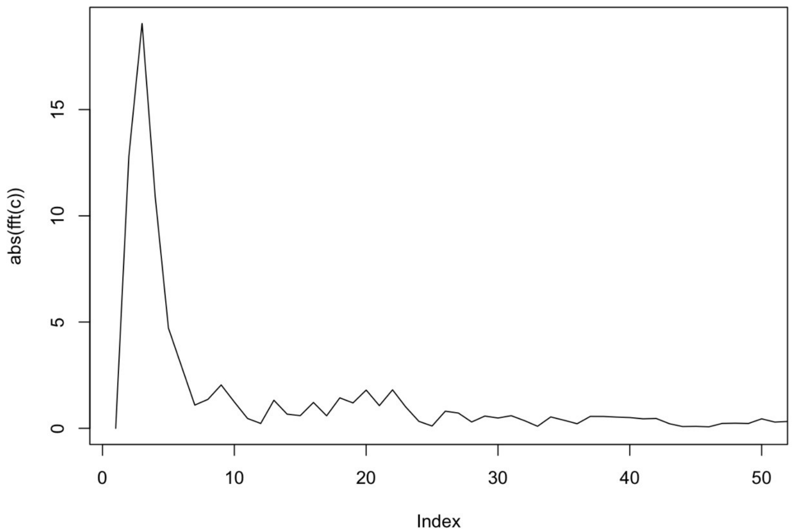

Figure 2, Fourier Transformation of the Residual Term from the Trend Model.

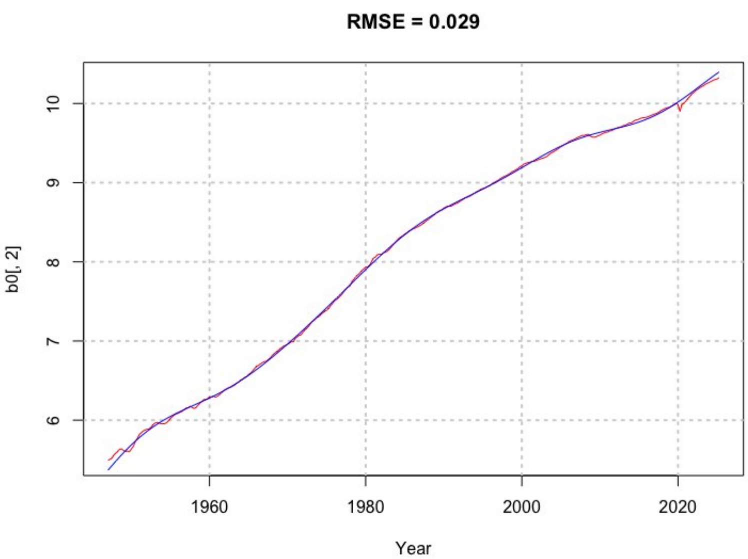

Figure 3, US Quarterly GDP (red line) and Model of both the Trend and Seasonality (blue line).

The partial autocorrelation function (PACF) of the residual term from the seasonality model is shown in Figure 4. It suggests that the GDP value of one quarter is correlated to those values three quarters prior. These contributing quarters are then included into the autoregression model of the residual term. Such a small number of quarters that are correlated to the later GDP growth suggests that contrary to the common belief (aka "January Barometer"), a single quarter growth at the beginning of a year may not be sufficient to judge the growth potential for the rest of the year.

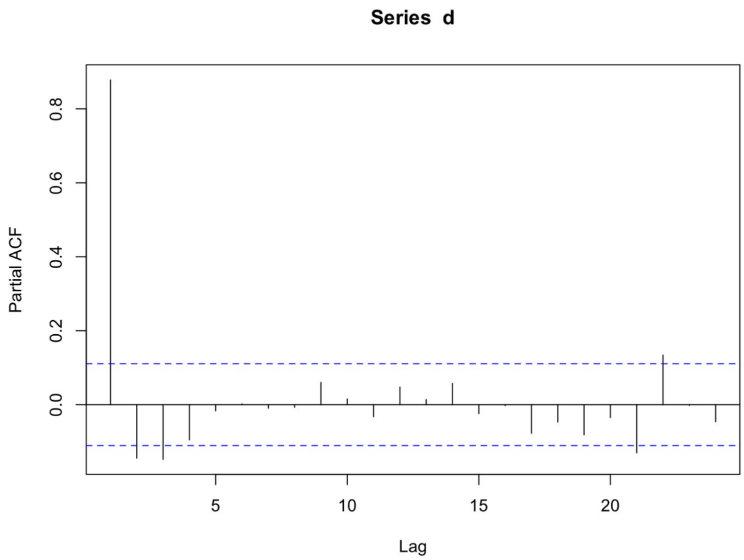

Figure 4, Partial Autocorrelation Function of the Residual Term (d) from the Seasonality Model.

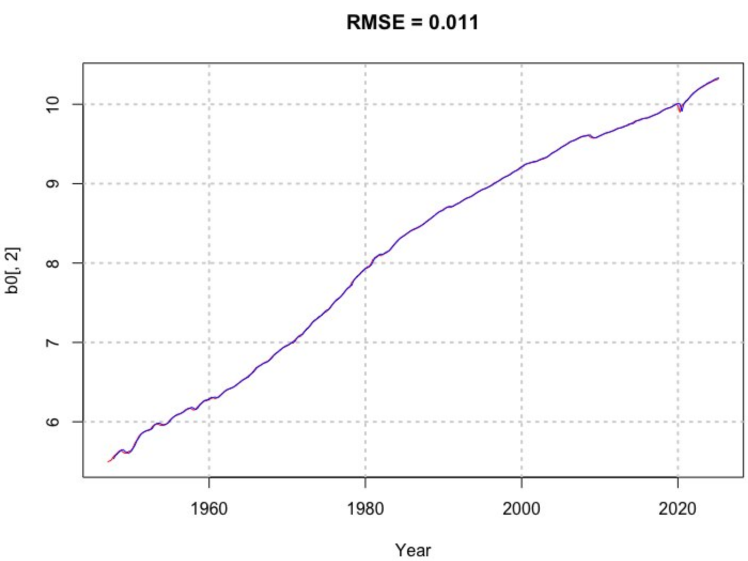

Figure 5, Combined Model with Trend, Seasonality, and Autoregression Components.

The combined model is shown in Figure 5. The RMSE of the combined model further reduces from 0.029 to 0.011. A closeup view of the model is also shown in Figure 6 for the GDP data (converted back from the logarithmic representation) for the recent years.

Figure 6, A Closeup View of Figure 5 of Actual Measurements (red line) and Model Prediction (blue line)

Concluding Remarks

The logarithmic representation of the US GDP is surprisingly well modeled using an extremely small number of model parameters (total 12 parameters over 80 years of growth data). These are 3 (linear and quadratic coefficients plus the intercept) for the trend model, 6 frequency components for the seasonality model, and 3 lagged quarters for the autoregression model. The fit between the model and the actual GDP data matches almost precisely (see Figure 5) except for the quarters during the 2020 pandemic. The recent data, however, show a small positive deviation of the model from the actual GDP numbers, suggesting that the US economy could grow slightly faster in view of the historical precedence and the "learnings" of the model.