Indexed Annuity Construction
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The consensus for a long term investment is to acquire an index mutual fund or ETF and hold it forever. The most popular index is US SP500 which has demonstrated an annual growth rate of 7% over a long period of time. However, from time to time, the investor may experience market gyration and feel psychological pain during a sharp market drawdown. To ease this investment dilemma, the insurance industry has been offering a product called fixed indexed annuity (FIA) that promises tracking a market index performance with no loss of investment capital on an annual basis. The word "fixed" implies a maximum cap on the index gain, *e.g.*, 10% if the index grows more than 10% in a year. Understanding this arrangement is important for an endowment fund or a pension fund that requires steady cash outlay to support its day-to-day operations irrespective of the market conditions. This article explains the method of constructing such an investment vehicle.

Fixed Income Investments (Bonds)

To preserve the investment capital, a major portion of the investment must be kept in a low risk or risk free asset, possibly earning a steady interest or dividend payment throughout the period. This can be a money market fund or high yield bond fund. The other, much smaller, portion of the investment is used to pay a "premium" for a futures contract that tracks an index performance. The premium portion (p) is dictated by the dividend yield (y) of the fixed income investment and its term (T):

$$p = \frac{(1+y)^T - 1}{(1+y)^T}$$

A high yield fixed income investment allows a greater portion of the total investment to be allocated for the futures contract. For example, at 4% interest rate, the available premium is 3.8% of the investment capital. This amount increases to 7.4% if the fixed income investment is a high-yield bond fund with 8% annualized dividend payment. This rule ensures that the investment capital is preserved at all times even when the futures contract is a complete loss in a given year.

Futures Contracts (Options)

A futures contract guarantees a specific price (strike price) of a product at a future date. The premium (C) for such a contract is the difference between today's price (S_0) and the discounted strike price (K). At continuous time compounding of the interest (r) through the contract term (T):

$$C = S_0 - Ke^{-rT}$$

If the futures contract is structured as an index option, *i.e.*, it provides the right, but not the obligation, of exercising the contract at it maturity, the payout of the contract becomes:

$$P = max\left\{S_T - K, 0\right\}$$

The contact will pay the difference between the index price at the maturity (S_T) and the strike price if the index price is higher than the strike price, or nothing if the index price is lower than the strike price. Therefore, during a good year, this arrangement provides a substantial leverage of the contract premium that locks a sizable profit without actually owning the index (which would require a large sum of money).

In order to track the index performance, the number of futures contracts (m) must be the same number of shares as if the total amount of the investment (assuming 1 unit) were used to purchase the index at its original share price (S_0):

$$m = \frac{1}{S_0}$$

The total cost of the futures contracts must be afforded by the earnings from the fixed income allocated for the premium:

$$mC = 1 - \frac{K}{S_0} e^{-rT} \leq \frac{(1+y)^T - 1}{(1+y)^T}$$

With the help of the Black-Scholes option price formula that accounts for the stochastic nature of the index price movements, the above relation can be more accurately expressed as follows:

$$N(d_1) - \frac{K}{S_0} N(d_2) e^{-rT} \leq \frac{(1+y)^T - 1}{(1+y)^T}$$

where $N(x)$ is the cumulative standard normal distribution, d_1 and d_2 are:

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}$$

where σ is the volatility of the index annualized return.

The SP500 1-year call option premium at the current 4% risk-free interest rate can be computed using the above formula while ignoring the transaction cost and assuming possible fractional contracts. To fully track the index performance, the strike price must be the same as the index price at the time of the call option contract (*i.e.*, $K = S_0$). As such, the required premium is approximately 9% of the total asset. However, the earnings from the fixed asset allocated for paying the premium is governed by the dividend yield. To generate an equivalent 9% return, the fixed asset must earn a dividend yield of 10%. This may not be possible on the current bond market with acceptable default risk of a bond. For example, the dividend yield of high yield corporate bonds (*aka*, junk bonds) is 5.9% which corresponds to only 5.5% total asset available for a call option premium payment. There is a 3.5% short fall to meet the call option premium requirement.

Fixed Indexed Annuity

The fixed indexed annuity offered by insurance companies simultaneously enters into two, not one, call option contracts to overcome this problem. First, the annuity purchases a SP500 1-year call option at the same strike price as the index price. Second, it sells a SP500 1-year call option at a strike price 10% ~ 15% higher than the index price. This arrangement (termed a vertical spread call

option) lows the total premium required to track the index performance but limits the maximum index growth to 10% ~ 15%. The premium requirement is shown in Figure 1. Selling an option contract at a strike price 15% higher than the index price, the required premium becomes 5.5% of the total asset instead of 9% (see the red line). Most insurance products, however, keep the maximum cap to 10% so that only 4.5% total asset is required to be used for the option contract premium. The difference between 4.5% and 5.5% can be used either to secure a higher investment grade bond, offset the transaction cost of the option contracts, or help pay the administrative cost.

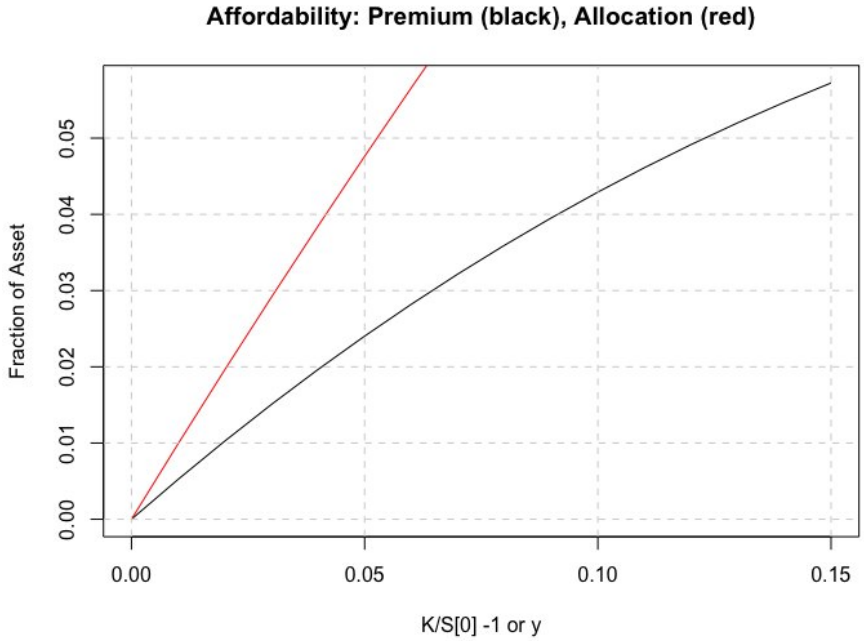

Figure 1. Option Premium with Vertical Spread and Required Allocation of Total Asset for the Premium Payment at Given Bond Dividend Yield

Simulated Performances

Based on the above discussion, the fixed indexed annuity tracks the index performance with two exceptions: 1) if the index annual growth is negative, the annuity growth becomes zero and 2) if the index annual growth is above 10% the annuity growth remains at 10%. Figure 2 shows its performance (the blue line). Indeed, the annuity never experienced a loss during the 20 year period, but it underperformed the SP500 at the end of 20 year period.

Also in Figure 2, another fixed indexed annuity with 15% performance cap is shown as the red line. It too did not experience a losing year, but outperformed SP500 at the end of the 20 year period. It should be noted that to pay for the higher call option premium, the annuity with 15% cap may have to rely on a lower grade bond fund to cover the extra cost. This can be a higher risk than the annuity with a 10% performance cap.

Alternatively, one may choose to allocate the required 9% asset every year to pay for the high option premium without a performance cap on the annuity. In this way, the annuity will experience a loss during a bad year, but the loss is limited at -3.5% of the total asset as the difference between the bond yield earnings and the option premium payment. The simulated performance is shown in Figure 3.

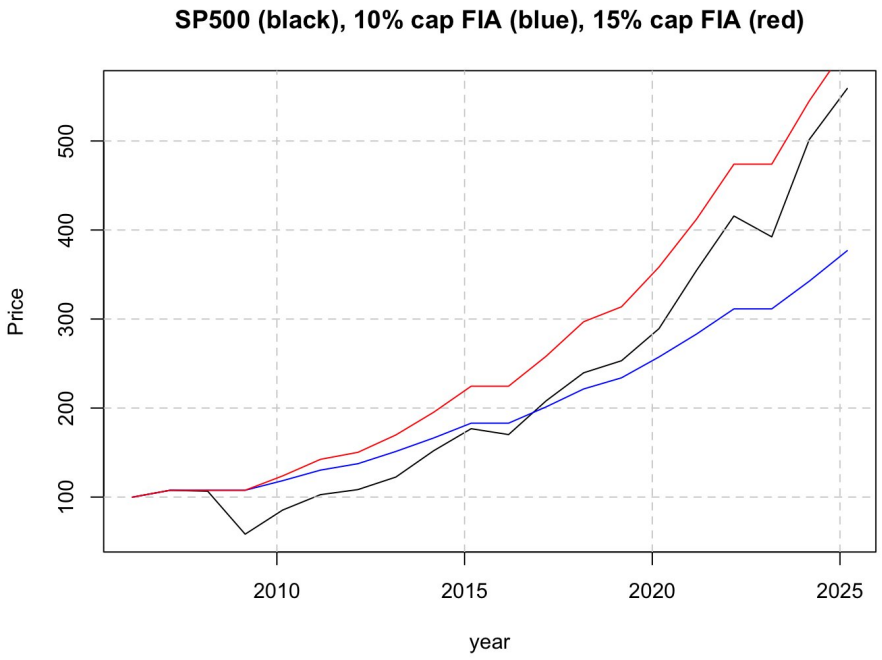

Figure 2. Simulated Performance of Fixed Indexed Annuities (FIA). SP500 – Black, FIA with 10% Cap – Blue, and FIA with 15% Cap - Red

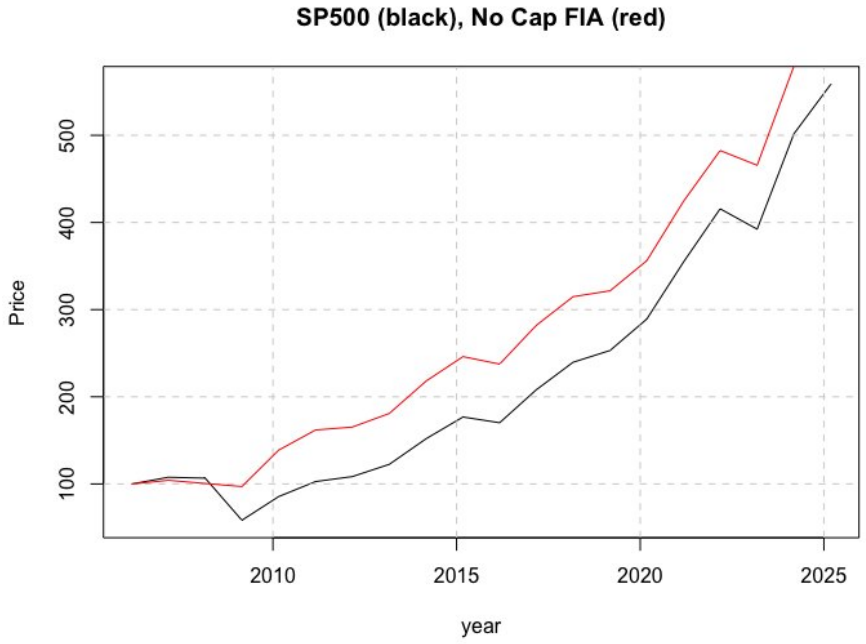

Figure 3. Simulated Performance of Indexed Annuities without Caps. SP500 – Black and FIA without Cap - Red