Anand Viscoplasic Model

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Anand's model is a unified viscoplastic constitutive model developed by L. Anand and colleagues in the 1980s. Unlike traditional plasticity models that separate rate-independent plasticity from rate-dependent creep, Anand's model uses a single scalar internal variable, the deformation resistance s, to represent the isotropic resistance to the inelastic flow. This makes it particularly suitable for modeling solder alloys, which exhibit significant rate-dependent behavior even at room temperature due to their low homologous temperature.

The Anand's model is based on several key assumptions:

1. *Small Deformation*: The model assumes infinitesimal deformations, allowing for additive decomposition of the strain rate tensor.
2. *Unified Viscoplasticity*: There is no explicit yield surface. Inelastic flow (creep/plasticity) occurs at all non-zero stress levels, governed by a single flow equation.
3. *Isotropic Hardening*: The internal state variable s (deformation resistance) is scalar, implying that the material hardens isotropically (no kinematic hardening or *Bauschinger effect*).
4. *Thermally Activated Flow*: The inelastic strain rate is controlled by thermal activation, typically modeled via an Arrhenius-type equation.
5. *Saturation*: The deformation resistance s saturates to a steady-state value s^* at a constant strain rate and temperature, representing the balance between work hardening and dynamic recovery.

Flow Equation

The equation can be derived from the *Orowan equation that* relates the plastic shear strain rate of a material to the mobile dislocation density, the magnitude of the *Burgers* vector, and the average dislocation velocity:

$$\dot{\varepsilon} = \rho b v$$

where ρ is mobile dislocation density, b is *Burgers* vector, and v is dislocation velocity. Dislocation velocity in crystalline materials is often described by a nonlinear, hyperbolic sine relationship with applied shear stress, particularly in scenarios where at low stresses the dislocation motion is slow and thermally assisted:

$$v \propto \exp\left(-\frac{Q}{RT}\right)\sinh\left(\frac{\tau V}{kT}\right)$$

Here, τ is shear stress and V is activation volume. In Anand's model, the stress is normalized by the deformation resistance s (which scales with ρ), and the hyperbolic sine function captures the transition from power-law creep (low stress) to exponential creep (high stress).

The inelastic strain rate $\dot{\varepsilon}_{in}$ is found to be governed by the hyperbolic sine law:

$$\dot{\varepsilon}_{in} = A\exp\left(-\frac{Q}{RT}\right)\left[\sinh\left(\xi\frac{\sigma}{s}\right)\right]^{1/m}$$

where σ is the true stress, T absolute temperature, s deformation resistance, an internal variable, A the pre-exponential factor, representing the frequency of atomic vibrations or the density of mobile dislocations, Q the activation energy, representing the energy barrier for dislocation motion or diffusion, R the universal gas constant (8.314 J/mol·K), ξ stress multiplier scaling the applied stress relative to the deformation resistance, m a strain rate sensitivity exponent determining how sensitive the flow stress is to changes in strain rate.

Evolution Equation for Deformation Resistance

The evolution of the deformation resistance s accounts for strain hardening and dynamic recovery:

$$\dot{s} = \left\{ h_0 \left(1 - \frac{s}{s^*}\right)^a \right\} \dot{\varepsilon}_{in}$$

The saturation value s^* is itself a function of strain rate and temperature:

$$s^* = \hat{s} \left[\frac{\dot{\varepsilon}_{in}}{A} \exp\left(\frac{Q}{RT}\right) \right]^n$$

where h_0 is hardening/softening constant, representing the initial hardening modulus, a the strain rate sensitivity of hardening that controls how quickly the hardening rate decays with deformation, $\hat{s}$ the coefficient for saturation value, n the strain rate sensitivity for saturation value that defines how s^* varies with the *Zener-Hollomon parameter* which describes the combined effect of temperature and strain rate on the plastic deformation behavior of metals.

The evolution equation is phenomenological, designed to ensure that the deformation resistance s asymptotically approaches a steady-state value s^* at various stages:

1. *Initial Hardening*: At the onset of inelastic flow ($s \ll s^*$), the term $(1 - s/s^*) \approx 1$. Thus, $\dot{s} \approx h_0 \dot{\varepsilon}_{in}$. The parameter h_0 governs the initial work hardening rate.
2. *Dynamic Recovery*: As deformation progresses, s increases. The term $(1 - s/s^*)$ decreases, reducing the hardening rate. This represents dynamic recovery, where dislocation annihilation balances dislocation generation.
3. *Saturation*: When $s = s^*$, the hardening rate becomes zero $(\dot{s} = 0)$. The material reaches a steady-state flow stress.
4. *Steady-State Stress*: Substituting $s = s^*$ into the flow equation yields the steady-state stress σ^*:

$$\dot{\varepsilon}_{in} = A \exp\left(-\frac{Q}{RT}\right) \left[\sinh\left(\xi \frac{\sigma^*}{s^*}\right) \right]^{1/m}$$

Application to Lead-Free Solder Alloys

Lead-free solders, such as Sn-3.0Ag-0.5Cu (SAC305) and Sn-3.5Ag, are critical for electronic packaging reliability. Their thermal-mechanical performance is characterized by Anand's model. At operational temperatures (*e.g.*, 25°C to 125°C), the melting point of SAC solders is approximately 217°C. This results in a homologous temperature $T/T_m > 0.6$. At such temperatures, time-dependent deformation (creep) dominates. Anand's unified model captures this without needing to separate "plasticity" from "creep."

During thermal cycling, solder joints experience varying strain rates due to the mismatch in Coefficient of Thermal Expansion (CTE) between the silicon chip and the printed circuit board (PCB). Anand's model accurately predicts:
- *Low Strain Rate (Creep)*: At high temperatures and slow ramp rates, the model predicts low flow stress due to dynamic recovery (s^* decreases).
- *High Strain Rate (Plasticity)*: At low temperatures or fast ramp rates, the model predicts high flow stress due to higher s^* and rate sensitivity.

Anand's model is the standard viscoplastic material model used in commercial FEA software (*e.g.*, ANSYS, Abaqus) for solder joint reliability simulations. The derivation above results in a set of differential equations that are integrated locally at each integration point during a simulation.

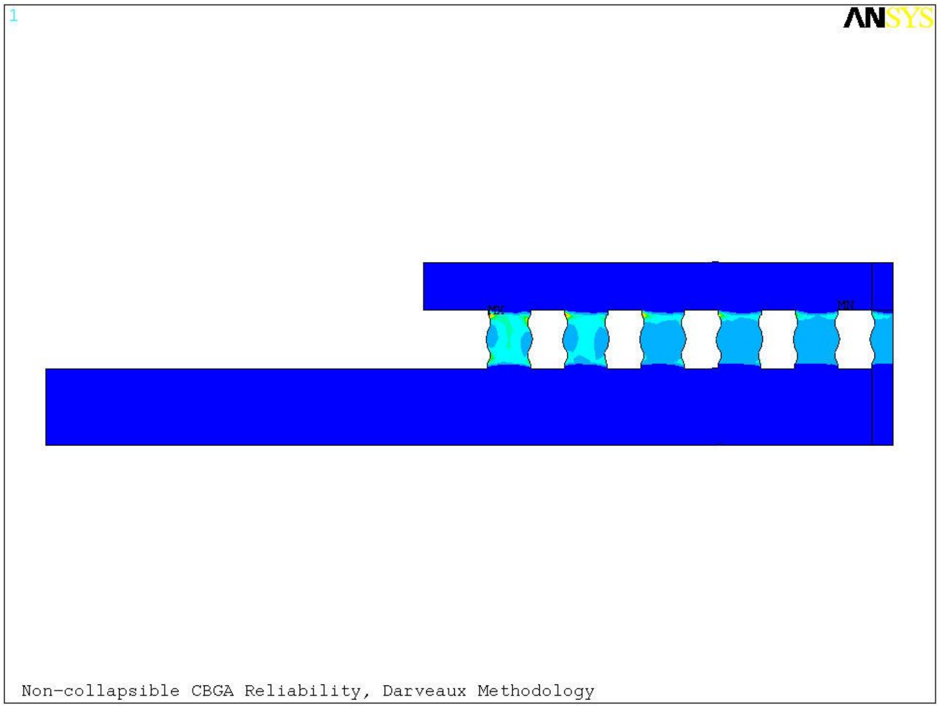

Figure 1. Anand Model used for CBGA Solder Joint Simulation

Accelerated Thermal Cycling

Anand's model is used to simulate accelerated thermal cycle (ATC) conditions (*e.g.,* -40°C to 125°C). The simulation outputs inelastic strain energy density per cycle (ΔW), which is correlated to the *Coffin-Manson* relationship to predict the number of cycles to failure. Figure 1 shows an example.

The parameters ($A, Q, \xi, m, h_0, \hat{s}, n, a$) are typically calibrated using a combination of:
- Constant strain rate tests (e.g., 0.0001/s to 0.1/s) at various temperatures.
- Creep tests (constant load) at various temperatures.
- Stress relaxation tests.

For SAC305, typical values show a high activation energy ($Q \approx 60 - 80\, kJ/mol$), indicating diffusion-controlled deformation, which is consistent with the dominant creep mechanism in β - Sn.

Despite its widespread use, the application of the standard Anand model has limitations when applied to lead-free solders:

- *Microstructural Evolution*: It does not account for coarsening of Ag_3Sn intermetallics or recrystallization, which significantly affect long-term reliability.
- *Anisotropy*: β-Sn is highly anisotropic. Anand's isotropic assumption can lead to inaccuracies in single-crystal or textured solder joints.
- *Damage Accumulation*: The model is purely mechanical; it does not inherently include damage or crack initiation. It must be coupled with a separate fatigue model (e.g., *Darveaux's model*) to predict failure.

Concluding Remarks

Anand's creep equation provides a robust, unified framework for describing the rate-dependent, temperature-dependent plastic flow of lead-free solder alloys. By deriving the flow equation from thermal activation theory and the evolution equation from saturation hardening dynamics, the model successfully decouples the material's response into rate-dependent hardening and steady-state behavior. Its implementation in FEA is critical for predicting the thermomechanical fatigue life of electronic assemblies, making it an indispensable tool in the microelectronics reliability field.

Appendix – Excerpt of an ANSYS Script

```
!######### Solder SN60, MAT 2, ET 2
MP,EX,2,-1.007e6,2.2e4  !modulus and temp coefficient, use K instead of C
MP,ALPX,2,25.0E-6       !CTE
MP,REFT,2,zs_temp       !zero stress temperaure
TB,ANAND,2
TBDATA,1,8.17E3         !s0
TBDATA,2,1.083e4        !Q/k
TBDATA,3,8.94e8         !A
TBDATA,4,11.0           !eta
TBDATA,5,0.303          !m
TBDATA,6,3.83e5         !h0
TBDATA,7,11.66e3        !s_hat
TBDATA,8,0.0231         !n
TBDATA,9,1.34           !a
MP,DENS,2,1

!########### TEMPERATURE CYCLES
/SOLU
ANTYPE,TRANS
NLGEOM,ON                 !large deformation on
KBC,0                     !ramp loading for all temperature changes
AUTOTS,ON
DELTIM,T_STEP
OUTRES,ESOL,LAST          !save all element solution
LNSRCH, ON

BFUNIF,TEMP,en_temp       !stay at en_temp
TIME,(zs_temp-en_temp)/rate
LSWRITE

LSSOLVE,1,1,1
FINISH
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