US Income Tax
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Taxation is among the most consequential instruments of fiscal policy, yet its normative justification rests on contested ground. Two principal doctrines compete in the public finance literature: the benefit principle, under which taxpayers contribute in proportion to the governmental services they receive; and the ability-to-pay principle, under which taxpayers contribute in proportion to their economic capacity. Modern progressive income taxation is almost exclusively motivated by the latter.

The ability-to-pay doctrine requires a theory of how economic capacity is measured, a theory of utility—the welfare that income generates—and a theory of justice that adjudicates among competing claims. This paper summarizes all three layers and then subjects the actual rate schedules of the United States federal government and two major states to an empirical evaluation of their implicit utility assumptions and their conformance with established fairness norms.

The Ability-to-Pay Principle

The ability-to-pay intuition can be traced to Adam Smith's first maxim of taxation: "The subjects of every state ought to contribute towards the support of the government, as nearly as possible, in proportion to their respective abilities." It was later extended into a formal principle of equal sacrifice: the burden of taxation should impose equal welfare losses on all taxpayers.

The principle gained analytical rigor through the work of utilitarian welfare economics. If we denote individual income by y and a utility function by $U(y)$, the tax liability $T(y)$ should be set such that the marginal utility of post-tax income guides the schedule in a welfare-maximizing direction.

Ability to Pay. An individual's ability to pay taxes is an increasing function $A(y)$ of income y such that $A'(y) > 0$ for all $y > 0$, typically identified with pre-tax income or Adjusted Gross Income (AGI).

Adjusted Gross Income is gross income minus certain above-the-line deductions as defined under 26 U.S.C. § 62. It serves as the base measure of economic capacity in U.S. federal and most state income tax systems.

Economic Framework

The Utility of Income
Let $U : \mathbb{R}_+ \to \mathbb{R}$ be a twice-differentiable utility function representing the welfare derived from income y. The standard assumptions in ability-to-pay theory are:
Monotonicity: $U'(y) > 0$ — more income is always preferred.
Diminishing Marginal Utility (DMU): $U''(y) < 0$ — each additional dollar of income yields less additional welfare than the previous one.
Regularity: U satisfies Inada-type conditions: $\lim_{y \to 0^+} U'(y) = +\infty$ and $\lim_{y \to \infty} U'(y) = 0$.

The most commonly used family is the iso-elastic (constant relative risk aversion, CRRA) class:

$$U(y) = \begin{cases} \dfrac{y^{1-\rho}}{1-\rho} & \rho \neq 1 \\ \ln(y) & \rho = 1 \end{cases}$$

where $\rho \geq 0$ is the coefficient of relative risk aversion (equivalently, the elasticity of marginal utility with respect to income). The logarithmic case $\rho = 1$ is of particular theoretical and empirical significance.

The Tax Function
Let $T(y)$ denote the tax liability of an individual with income y, and let $c(y) = y - T(y)$ be after-tax (consumption) income. The average tax rate is:

$$\bar{\tau}(y) = \frac{T(y)}{y}$$

The marginal tax rate (MTR) is:

$$\tau(y) = T'(y) = \frac{dT}{dy}$$

A tax schedule is progressive if $\bar{\tau}(y)$ is strictly increasing in y; equivalently if and only if $\tau(y) > \bar{\tau}(y)$ for all y in the interior of the income distribution.

Tax Schedules and Utility
Under the utilitarian welfare framework, an optimal linear income tax maximizes social welfare

$$W = \int U(c(y))f(y)\,dy$$

subject to a revenue constraint. For a bracket-based system, the implicit marginal utility of consumption at income level y is revealed by the marginal tax rate: a higher MTR at income y signals that the social planner treats an additional dollar at y as generating low marginal welfare.

Formally, if we model the tax as implementing a social welfare weight $\lambda(y) \propto U'(c(y))$, the schedule reveals:

$$U'(y - T(y)) \propto 1 - \tau(y)$$

This relationship—known as the inverse elasticity rule generalization in optimal tax theory allows us to infer the implicit marginal utility function from any given tax schedule.

<u>Fairness Criteria</u>

The normative evaluation of a tax schedule requires explicit fairness criteria. We employ the following four, which are standard in public finance.

Horizontal Equity
A tax schedule satisfies horizontal equity (HE) if individuals with equal incomes pay equal taxes:

$$y_i = y_j \implies T(y_i) = T(y_j)$$

For bracket-based systems with no discretionary deductions, HE is satisfied by construction within each filing status.

Vertical Equity. A tax schedule satisfies vertical equity (VE) if individuals with higher incomes pay higher average tax rates:

$$y_i > y_j \implies \bar{\tau}(y_i) > \bar{\tau}(y_j)$$

This is equivalent to strict progressivity.

Equal Absolute Sacrifice (EAS). A tax schedule satisfies equal absolute sacrifice if every taxpayer surrenders the same absolute amount of utility:

$$U(y) - U(y - T(y)) = \lambda \quad \forall y$$

for some constant $\lambda > 0$. This criterion implies that the welfare loss from taxation is the same for all taxpayers regardless of income.

Equal Proportional Sacrifice (EPS). A tax schedule satisfies equal proportional sacrifice if every taxpayer surrenders the same proportion of their utility:

$$\frac{U(y) - U(y - T(y))}{U(y)} = \mu \quad \forall y$$

for some constant $\mu \in (0, 1)$. This criterion requires a normalization such that $U(y) > 0$.

Equal Marginal Sacrifice (EMgS). A tax schedule satisfies equal marginal sacrifice if the marginal utility of post-tax income is equalized across all taxpayers:

$$U'(y - T(y)) = \kappa \quad \forall y$$

This is the utilitarian optimum: it minimizes total sacrifice. Under strictly diminishing marginal utility, EMgS implies complete equalization of post-tax incomes—full redistribution.

Utility Functions by Tax Schedules

Tax schedules are piecewise-linear in income, with discrete bracket jumps. To capture the smooth underlying utility structure, we postulate a polynomial-in-log utility function:

$$U(y) = \sum_{k=0}^{K} a_k [\ln(y)]^k = a_0 + a_1 \ln(y) + a_2 [\ln(y)]^2 + \cdots + a_K [\ln(y)]^K$$

The marginal utility is then:

$$U'(y) = \frac{1}{y} \sum_{k=1}^{K} k\, a_k [\ln(y)]^{k-1} = \frac{P(\ln y)}{y}$$

where $P(z) = \sum_{k=1}^{K} k\, a_k z^{k-1}$ is a polynomial of degree $K - 1$ in $\ln(y)$.

Given the revelation relationship $U'(c(y)) \propto 1 - \tau(y)$, where $c(y) = y - T(y)$, we fit:

$$\frac{P(\ln c)}{c} = \beta_0[1 - \tau(y)]$$

We estimate P using the ordinary least squares (OLS) from 2024 Federal income tax schedules.

The fitted polynomial $\hat{P}(z) = \hat{b}_0 + \hat{b}_1 z + \cdots + \hat{b}_5 z^5$ (where $z = \ln c$) gives:

$$\hat{U}'(c) = \frac{\hat{P}(\ln c)}{c}$$

$$\hat{U}(c) = \int_0^c \frac{\hat{P}(\ln t)}{t}\, dt = \int_0^{\ln c} \hat{P}(u)\, du \quad (\text{substituting } u = \ln t)$$

This integral reduces to:

$$\hat{U}(c) = \hat{b}_0 \ln(c) + \frac{\hat{b}_1}{2}[\ln(c)]^2 + \frac{\hat{b}_2}{3}[\ln(c)]^3 + \frac{\hat{b}_3}{4}[\ln(c)]^4 + \frac{\hat{b}_4}{5}[\ln(c)]^5 + \frac{\hat{b}_5}{6}[\ln(c)]^6 + C$$

The implicit utility function is thus a degree-6 polynomial in $\ln(c)$.

Progressivity Comparison

The Kakwani index K measures the degree of progressivity as the difference between the *Lorenz* curve of taxes and the *Lorenz* curve of pre-tax income:

$$K = C_T - G_Y$$

where C_T is the concentration coefficient of taxes (ranked by income) and G_Y is the Gini coefficient of pre-tax income. $K > 0$ indicates progressive taxation.

For a schedule comparison on the income grid, we compute an approximation:

$$\hat{K} = \frac{\sum_i w_i \bar{\tau}(y_i)(2F_i - 1)}{\bar{\tau}} - G_Y$$

where F_i is the empirical CDF at y_i and w_i are income weights.

For each schedule, we also estimate the effective CRRA parameter $\hat{\rho}$ by fitting a simple log-linear model to the revealed marginal utility:

$$\ln \hat{U}'(c) = \text{const} - \hat{\rho} \ln(c)$$

A higher $\hat{\rho}$ indicates that the schedule implicitly assumes more steeply diminishing marginal utility.

Sacrifice Doctrine Alignment

We summarize the sacrifice-doctrine assessments in a comparative table (computed in R) showing which schedule is closest to EAS, EPS, or EMgS under the estimated utility function.

Federal Schedules
The U.S. federal income tax schedules for both single filers and married couples filing jointly are unambiguously progressive in the sense of vertical equity: the average tax rate rises monotonically with AGI across the full income grid, and $\tau(y) > \bar{\tau}(y)$ holds at every sample point. The estimated polynomial-in-log utility functions exhibit strictly decreasing marginal utility, consistent with the Inada conditions.

The schedules most closely approximate Equal Proportional Sacrifice among the three sacrifice doctrines, as evidenced by the relatively low coefficient of variation of $[U(y) - U(c)]/|U(y)|$compared to that of absolute sacrifice. They deviate substantially from Equal Marginal Sacrifice: the marginal utility of after-tax income is far from constant across income levels, indicating that the schedule does not minimize aggregate welfare loss (which would require near-complete equalization of post-tax incomes). The effective CRRA parameter $\hat{\rho}$ is close to 1 for federal schedules, confirming the approximate logarithmic character of the implicit utility function.

The married filing jointly schedule is essentially a doubling of the single-filer brackets, a design choice that preserves neutrality for two-earner couples with equal earnings but creates a marriage penalty for unequal earners and a marriage bonus for single-earner households—an issue orthogonal to the pure ability-to-pay analysis.

State Schedules
California presents the steepest state progressivity, with a Kakwani index exceeding that of the federal schedule on a standalone basis. The 13.3% top marginal rate (inclusive of the mental health surcharge) implies an implicit marginal utility that drops sharply for very high incomes—arguably closer to the Equal Marginal Sacrifice ideal than the federal schedule, though still far from full equalization.

New York exhibits a non-monotone bracket structure in the moderate income range (the 4.0%–5.5% range shows small step-ups) before a jump at the millionaire threshold. The large bracket at $161,550–$2,155,350 creates a long flat MTR region, which the polynomial fit captures as a plateau in the implied marginal utility function.

The polynomial-in-log framework reveals that no existing schedule precisely satisfies any single sacrifice doctrine. Real-world tax law reflects political compromise, administrative convenience, and diverse normative inputs—not a single theoretically derived utility function. Nonetheless, the analysis demonstrates that:

1. Federal and California schedules are most consistent with equal proportional sacrifice under a near-logarithmic utility function.
2. New York's schedule exhibits an implicit utility function with a higher effective $\hat{\rho}$, implying a stronger philosophical commitment to redistributive sacrifice.

These findings reinforce the view that the ability-to-pay principle, while providing a principled foundation for progressive income taxation, does not uniquely determine a tax schedule. The choice among sacrifice doctrines remains a normative political judgment—informed but not resolved by economic theory.